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Part A

FORMULAS

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Section I: Elementary Constants, Products, Formulas

1 GREEK ALPHABET and SPECIAL CONSTANTS

GREEK ALPHABET

Greek name	Greek letter: Lower case	Greek letter: Capital
Alpha	α	A
Beta	β	B
Gamma	γ	Γ
Delta	δ	Δ
Epsilon	ϵ	E
Zeta	ζ	Z
Eta	η	H
Theta	θ	Θ
Iota	ι	I
Kappa	κ	K
Lambda	λ	Λ
Mu	μ	M

Greek name	Greek letter: Lower case	Greek letter: Capital
Nu	ν	N
Xi	ξ	Ξ
Omicron	o	O
Pi	π	Π
Rho	ρ	P
Sigma	σ	Σ
Tau	τ	T
Upsilon	υ	Υ
Phi	ϕ	Φ
Chi	χ	X
Psi	ψ	Ψ
Omega	ω	Ω

SPECIAL CONSTANTS

1.1 $\pi = 3.14159\ 26535\ 89793\ 23846\ 2643\ldots$

1.2 $e = 2.71828\ 18284\ 59045\ 23536\ 0287\ldots = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$
$= \text{natural base of logarithms}$

1.3 $\sqrt{2} = 1.41421\ 35623\ 73095\ 0488\ldots$

1.4 $\sqrt{3} = 1.73205\ 08075\ 68877\ 2935\ldots$

1.5 $\sqrt{5} = 2.23606\ 79774\ 99789\ 6964\ldots$

1.6 $\sqrt[3]{2} = 1.25992\ 1050\ldots$

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1.7 $\sqrt[3]{3} = 1.44224\ 9570\ldots$

1.8 $\sqrt[5]{2} = 1.14869\ 8355\ldots$

1.9 $\sqrt[5]{3} = 1.24573\ 0940\ldots$

1.10 $e^{\pi} = 23.14069\ 26327\ 79269\ 006\ldots$

1.11 $\pi^{e} = 22.45915\ 77183\ 61045\ 47342\ 715\ldots$

1.12 $e^{e} = 15.15426\ 22414\ 79264\ 190\ldots$

1.13 $\log_{10} 2 = 0.30102\ 99956\ 63981\ 19521\ 37389\ldots$

1.14 $\log_{10} 3 = 0.47712\ 12547\ 19662\ 43729\ 50279\ldots$

1.15 $\log_{10} e = 0.43429\ 44819\ 03251\ 82765\ldots$

1.16 $\log_{10} \pi = 0.49714\ 98726\ 94133\ 85435\ 12683\ldots$

1.17 $\log_e 10 = \ln 10 = 2.30258\ 50929\ 94045\ 68401\ 7991\ldots$

1.18 $\log_e 2 = \ln 2 = 0.69314\ 71805\ 59945\ 30941\ 7232\ldots$

1.19 $\log_e 3 = \ln 3 = 1.09861\ 22886\ 68109\ 69139\ 5245\ldots$

1.20 $\gamma = 0.57721\ 56649\ 01532\ 86060\ 6512\ldots$ = *Euler's constant*

$$= \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln n\right)$$

1.21 $e^{\gamma} = 1.78107\ 24179\ 90197\ 9852\ldots$ [see 1.20]

1.22 $\sqrt{e} = 1.64872\ 12707\ 00128\ 1468\ldots$

1.23 $\sqrt{\pi} = \Gamma(\frac{1}{2}) = 1.77245\ 38509\ 05516\ 02729\ 8167\ldots$
where Γ is the *gamma function* [see 25.1].

1.24 $\Gamma(\frac{1}{3}) = 2.67893\ 85347\ 07748\ldots$

1.25 $\Gamma(\frac{1}{4}) = 3.62560\ 99082\ 21908\ldots$

1.26 1 radian = $180°/\pi = 57.29577\ 95130\ 8232\ldots°$

1.27 $1° = \pi/180$ radians $= 0.01745\ 32925\ 19943\ 29576\ 92\ldots$ radians

2 SPECIAL PRODUCTS and FACTORS

2.1 $(x+y)^2 = x^2 + 2xy + y^2$

2.2 $(x-y)^2 = x^2 - 2xy + y^2$

2.3 $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

2.4 $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

2.5 $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

2.6 $(x-y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

2.7 $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

2.8 $(x-y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

2.9 $(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

2.10 $(x-y)^6 = x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$

The results 2.1 to 2.10 above are special cases of the *binomial formula* [see 3.3].

2.11 $x^2 - y^2 = (x-y)(x+y)$

2.12 $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

2.13 $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

2.14 $x^4 - y^4 = (x-y)(x+y)(x^2 + y^2)$

2.15 $x^5 - y^5 = (x-y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$

2.16 $x^5 + y^5 = (x+y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$

2.17 $x^6 - y^6 = (x-y)(x+y)(x^2 + xy + y^2)(x^2 - xy + y^2)$

2.18 $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$

2.19 $x^4 + 4y^4 = (x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)$

Some generalizations of the above are given by the following results where n is a positive integer.

2.20
$$\begin{aligned} x^{2n+1} - y^{2n+1} &= (x-y)(x^{2n} + x^{2n-1}y + x^{2n-2}y^2 + \cdots + y^{2n}) \\ &= (x-y)\left(x^2 - 2xy\cos\frac{2\pi}{2n+1} + y^2\right)\left(x^2 - 2xy\cos\frac{4\pi}{2n+1} + y^2\right) \\ &\quad \cdots \left(x^2 - 2xy\cos\frac{2n\pi}{2n+1} + y^2\right) \end{aligned}$$

2.21
$$\begin{aligned} x^{2n+1} + y^{2n+1} &= (x+y)(x^{2n} - x^{2n-1}y + x^{2n-2}y^2 - \cdots + y^{2n}) \\ &= (x+y)\left(x^2 + 2xy\cos\frac{2\pi}{2n+1} + y^2\right)\left(x^2 + 2xy\cos\frac{4\pi}{2n+1} + y^2\right) \\ &\quad \cdots \left(x^2 + 2xy\cos\frac{2n\pi}{2n+1} + y^2\right) \end{aligned}$$

2.22
$$\begin{aligned} x^{2n} - y^{2n} &= (x-y)(x+y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \cdots)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \cdots) \\ &= (x-y)(x+y)\left(x^2 - 2xy\cos\frac{\pi}{n} + y^2\right)\left(x^2 - 2xy\cos\frac{2\pi}{n} + y^2\right) \\ &\quad \cdots \left(x^2 - 2xy\cos\frac{(n-1)\pi}{n} + y^2\right) \end{aligned}$$

2.23
$$\begin{aligned} x^{2n} + y^{2n} &= \left(x^2 + 2xy\cos\frac{\pi}{2n} + y^2\right)\left(x^2 + 2xy\cos\frac{3\pi}{2n} + y^2\right) \\ &\quad \cdots \left(x^2 + 2xy\cos\frac{(2n-1)\pi}{2n} + y^2\right) \end{aligned}$$

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3 The BINOMIAL FORMULA and BINOMIAL COEFFICIENTS

FACTORIAL n

For $n = 1, 2, 3, \ldots$, *factorial n* or *n factorial* is denoted and defined by:

3.1 $$n! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$$

Zero factorial is defined by

3.2 $$0! = 1$$

Alternately, n factorial can be defined recursively by

$$0! = 1 \quad \text{and} \quad n! = n \cdot (n-1)!$$

Example: $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$,
$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 5 \cdot 4! = 5(24) = 120$,
$6! = 6 \cdot 5! = 6(120) = 720$

BINOMIAL FORMULA FOR POSITIVE INTEGRAL n

For $n = 1, 2, 3, \ldots$,

3.3 $$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \cdots + y^n$$

This is called the *binomial formula*. It can be extended to other values of n, and also to an infinite series [see 22.4].

Example:

(*a*) $(a-2b)^4 = a^4 + 4a^3(-2b) + 6a^2(-2b)^2 + 4a(-2b)^3 + (-2b)^4 = a^4 - 8a^3b + 24a^2b^2 - 32ab^3 + 16b^4$
Here $x = a$ and $y = -2b$.

(*b*) See Fig. 3-1(*a*).

BINOMIAL COEFFICIENTS

Formula 3.3 can be rewritten in the form

3.4 $$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \binom{n}{3}x^{n-3}y^3 + \cdots + \binom{n}{n}y^n$$

where the coefficients, called *binomial coefficients*, are given by:

3.5 $$\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} = \frac{n!}{k!(n-k)!} = \binom{n}{n-k}$$

Example: $\binom{9}{4} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4} = 126$, $\binom{12}{5} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 792$, $\binom{10}{7} = \binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120$

Note that $\binom{n}{r}$ has exactly r factors in both the numerator and the denominator.

The binomial coefficients may be arranged in a triangular array of numbers, called Pascal's triangle, as shown in Fig. 3-1(*b*). The triangle has the following two properties:

(1) The first and last number in each row is 1.

(2) Every other number in the array can be obtained by adding the two numbers appearing directly above it. For example:

$$10 = 4 + 6, \quad 15 = 5 + 10, \quad 20 = 10 + 10$$

Property (2) may be stated as follows:

3.6 $$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

$$\begin{aligned}
(a+b)^0 &= 1 \\
(a+b)^1 &= a + b \\
(a+b)^2 &= a^2 + 2ab + b^2 \\
(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
(a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \\
(a+b)^5 &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 \\
(a+b)^6 &= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6 \\
&\cdots\cdots
\end{aligned}$$

(*a*)

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  (10) 10   5   1
1   6  (15) (20) 15   6   1
.........................
```

(*b*)

Fig. 3-1

PROPERTIES OF BINOMIAL COEFFICIENTS

The following lists additional properties of the binomial coefficients:

3.7 $$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n$$

3.8 $$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots (-1)^n \binom{n}{n} = 0$$

3.9 $$\binom{n}{n} + \binom{n+1}{n} + \binom{n+2}{n} + \cdots + \binom{n+m}{n} = \binom{n+m+1}{n+1}$$

3.10 $$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = 2^{n-1}$$

3.11 $$\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots = 2^{n-1}$$

3.12 $$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$$

3.13 $$\binom{m}{0}\binom{n}{p} + \binom{m}{1}\binom{n}{p-1} + \cdots + \binom{m}{p}\binom{n}{0} = \binom{m+n}{p}$$

3.14 $$(1)\binom{n}{1} + (2)\binom{n}{2} + (3)\binom{n}{3} + \cdots + (n)\binom{n}{n} = n2^{n-1}$$

3.15 $$(1)\binom{n}{1} - (2)\binom{n}{2} + (3)\binom{n}{3} - \cdots (-1)^{n+1}(n)\binom{n}{n} = 0$$

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MULTINOMIAL FORMULA

Let $n_1, n_2, \ldots, n_r$ be nonnegative integers such that $n_1 + n_2 + \cdots + n_r = n$. Then the following expression, called a *multinomial coefficient*, is defined as follows:

3.16

$$\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1! n_2! \cdots n_r!}$$

Example: $\binom{7}{2, 3, 2} = \frac{7!}{2!3!2!} = 210, \quad \binom{8}{4, 2, 2, 0} = \frac{8!}{4!2!2!0!} = 420$

The name multinomial coefficient comes from the following formula:

3.17

$$(x_1 + x_2 + \cdots + x_p)^n = \sum \binom{n}{n_1, n_2, \ldots, n_r} x_1^{n_1} x_2^{n_2} \cdots x_r^{n_r}$$

where the sum, denoted by Σ, is taken over all possible multinomial coefficients.

4 COMPLEX NUMBERS

DEFINITIONS INVOLVING COMPLEX NUMBERS

A complex number z is generally written in the form

$$z = a + bi$$

where a and b are real numbers and i, called the *imaginary unit*, has the property that $i^2 = -1$. The real numbers a and b are called the *real* and *imaginary parts* of $z = a + bi$, respectively.

The *complex conjugate* of z is denoted by $\bar{z}$; it is defined by:

$$\overline{a + bi} = a - bi$$

Thus $a + bi$ and $a - bi$ are conjugates of each other.

EQUALITY OF COMPLEX NUMBERS

4.1 $a + bi = c + di$ if and only if $a = c$ and $b = d$

ARITHMETIC OF COMPLEX NUMBERS

Formulas for the addition, subtraction, multiplication, and division of complex numbers follow:

4.2 $(a + bi) + (c + di) = (a + c) + (b + d)i$

4.3 $(a + bi) - (c + di) = (a - c) + (b - d)i$

4.4 $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

4.5 $\dfrac{a + bi}{c + di} = \dfrac{a + bi}{c + di} \cdot \dfrac{c - di}{c - di} = \dfrac{ac + bd}{c^2 + d^2} + \left(\dfrac{bc - ad}{c^2 + d^2}\right)i$

Note that the above operations are obtained by using the ordinary rules of algebra and replacing i^2 by -1 wherever it occurs.

Example: Suppose $z = 2 + 3i$ and $w = 5 - 2i$. Then

$$z + w = (2 + 3i) + (5 - 2i) = 2 + 5 + 3i - 2i = 7 + i$$

$$zw = (2 + 3i)(5 - 2i) = 10 + 15i - 4i - 6i^2 = 16 + 11i$$

$$\bar{z} = \overline{2 + 3i} = 2 - 3i \text{ and } \bar{w} = \overline{5 - 2i} = 5 + 2i$$

$$\frac{w}{z} = \frac{5 - 2i}{2 + 3i} = \frac{(5 - 2i)(2 - 3i)}{(2 + 3i)(2 - 3i)} = \frac{4 - 19i}{13} = \frac{4}{13} - \frac{19}{13}i$$

COMPLEX PLANE

Real numbers can be represented by the points on a line, called the *real line*, and, similarly, complex numbers can be represented by points in the plane, called the *Argand diagram* or *Gaussian plane* or, simply, the *complex plane*. Specifically, we let the point (a, b) in the plane represent the complex number $z = a + bi$. For example, the point P in Fig. 4-1(a) represents the complex number $z = -3 + 4i$. The complex number can also be interpreted as a vector from the origin O to the point P.

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The *absolute value* of a complex number $z = a + bi$, written $|z|$, is defined as follows:

4.6 $$|z| = \sqrt{a^2 + b^2} = \sqrt{z\bar{z}}$$

We note $|z|$ is the distance from the origin O to the point z in the complex plane.

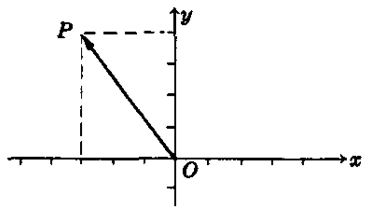

Fig. 4-1

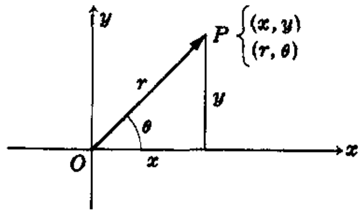

Fig. 4-2

POLAR FORM OF COMPLEX NUMBERS

The point P in Fig. 4-2 with coordinates (x, y) represents the complex number $z = x + iy$. The point P can also be represented by *polar coordinates* (r, θ). Since $x = r\cos\theta$ and $y = r\sin\theta$, we have

4.7 $$z = x + iy = r(\cos\theta + i\sin\theta)$$

called the *polar form* of the complex number. We often call $r = |z| = \sqrt{x^2 + y^2}$ the *modulus* and θ the *amplitude* of $z = x + iy$.

MULTIPLICATION AND DIVISION OF COMPLEX NUMBERS IN POLAR FORM

4.8 $$[r_1(\cos\theta_1 + i\sin\theta_1)][r_2(\cos\theta_2 + i\sin\theta_2)] = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

4.9 $$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

DE MOIVRE'S THEOREM

For any real number p, De Moivre's theorem states that:

4.10 $$[r(\cos\theta + i\sin\theta)]^p = r^p(\cos p\theta + i\sin p\theta)$$

ROOTS OF COMPLEX NUMBERS

Let $p = 1/n$ where n is any positive integer. Then 4.10 can be written

4.11 $$[r(\cos\theta + i\sin\theta)]^{1/n} = r^{1/n}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right)$$

where k is any integer. From this formula, all the n nth roots of a complex number can be obtained by putting $k = 0, 1, 2, \ldots, n - 1$.

5 SOLUTIONS of ALGEBRAIC EQUATIONS

QUADRATIC EQUATION: $ax^2 + bx + c = 0$

5.1 Solutions: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If a, b, c are real and if $D = b^2 - 4ac$ is the *discriminant*, then the roots are

(i) real and unequal if $D > 0$

(ii) real and equal if $D = 0$

(iii) complex conjugate if $D < 0$

5.2 If x_1, x_2 are the roots, then $x_1 + x_2 = -b/a$ and $x_1 x_2 = c/a$.

CUBIC EQUATION: $x^3 + a_1 x^2 + a_2 x + a_3 = 0$

Let $$Q = \frac{3a_2 - a_1^2}{9}, \quad R = \frac{9a_1 a_2 - 27a_3 - 2a_1^3}{54},$$
$$S = \sqrt[3]{R + \sqrt{Q^3 + R^2}}, \quad T = \sqrt[3]{R - \sqrt{Q^3 + R^2}}$$

where $ST = -Q$.

5.3 Solutions: $$\begin{cases} x_1 = S + T - \frac{1}{3}a_1 \\ x_2 = -\frac{1}{2}(S+T) - \frac{1}{3}a_1 + \frac{1}{2}i\sqrt{3}(S-T) \\ x_3 = -\frac{1}{2}(S+T) - \frac{1}{3}a_1 - \frac{1}{2}i\sqrt{3}(S-T) \end{cases}$$

If a_1, a_2, a_3 are real and if $D = Q^3 + R^2$ is the *discriminant*, then

(i) one root is real and two complex conjugate if $D > 0$

(ii) all roots are real and at least two are equal if $D = 0$

(iii) all roots are real and unequal if $D < 0$.

If $D < 0$, computation is simplified by use of trigonometry.

5.4 Solutions if $D < 0$: $$\begin{cases} x_1 = 2\sqrt{-Q}\cos(\frac{1}{3}\theta) - \frac{1}{3}a_1 \\ x_2 = 2\sqrt{-Q}\cos(\frac{1}{3}\theta + 120^\circ) - \frac{1}{3}a_1 \\ x_3 = 2\sqrt{-Q}\cos(\frac{1}{3}\theta + 240^\circ) - \frac{1}{3}a_1 \end{cases} \quad \text{where } \cos\theta = R/\sqrt{-Q^3}$$

5.5 $$x_1 + x_2 + x_3 = -a_1, \quad x_1x_2 + x_2x_3 + x_3x_1 = a_2, \quad x_1x_2x_3 = -a_3$$

where x_1, x_2, x_3 are the three roots.

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QUARTIC EQUATION: $x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$

Let y_1 be a real root of the following cubic equation:

5.6 $$y^3 - a_2y^2 + (a_1a_3 - 4a_4)y + (4a_2a_4 - a_3^2 - a_1^2a_4) = 0$$

The four roots of the quartic equation are the four roots of the following equation:

5.7 $$z^2 + \tfrac{1}{2}\{a_1 \pm \sqrt{a_1^2 - 4a_2 + 4y_1}\}z + \tfrac{1}{2}\{y_1 \mp \sqrt{y_1^2 - 4a_4}\} = 0$$

Suppose that all roots of 5.6 are real; then computation is simplified by using the particular real root that produces all real coefficients in the quadratic equation 5.7.

5.8 $$\begin{cases} x_1 + x_2 + x_3 + x_4 = -a_1 \\ x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1 + x_1x_3 + x_2x_4 = a_2 \\ x_1x_2x_3 + x_2x_3x_4 + x_1x_2x_4 + x_1x_3x_4 = -a_3 \\ x_1x_2x_3x_4 = a_4 \end{cases}$$

where x_1, x_2, x_3, x_4 are the four roots.

6 CONVERSION FACTORS

Length

1 kilometer (km)	= 1000 meters (m)	1 inch (in.)	= 2.540 cm
1 meter (m)	= 100 centimeters (cm)	1 foot (ft)	= 30.48 cm
1 centimeter (cm)	= 10^{-2} m	1 mile (mi)	= 1.609 km
1 millimeter (mm)	= 10^{-3} m	1 mil	= 10^{-3} in.
1 micron (μ)	= 10^{-6} m	1 centimeter	= 0.3937 in.
1 millimicron ($m\mu$)	= 10^{-9} m	1 meter	= 39.37 in.
1 angstrom (A)	= 10^{-10} m	1 kilometer	= 0.6214 mile

Area

1 square meter (m^2)	= 10.76 ft^2	1 square mile (mi^2)	= 640 acres
1 square foot (ft^2)	= 929 cm^2	1 acre	= 43,560 ft^2

Volume

1 liter (l) = 1000 cm^3 = 1.057 quart (qt) = 61.02 in^3 = 0.03532 ft^3
1 cubic meter (m^3) = 1000 l = 35.32 ft^3
1 cubic foot (ft^3) = 7.481 U.S. gal = 0.02832 m^3 = 28.32 l
1 U.S. gallon (gal) = 231 in^3 = 3.785 l; 1 British gallon = 1.201 U.S. gallon = 277.4 in^3

Mass

1 kilogram (kg) = 2.2046 pounds (lb) = 0.06852 slug; 1 lb = 453.6 gm = 0.03108 slug
1 slug = 32.174 lb = 14.59 kg

Speed

1 km/hr = 0.2778 m/sec = 0.6214 mi/hr = 0.9113 ft/sec
1 mi/hr = 1.467 ft/sec = 1.609 km/hr = 0.4470 m/sec

Density

1 gm/cm^3 = 10^3 kg/m^3 = 62.43 lb/ft^3 = 1.940 slug/ft^3
1 lb/ft^3 = 0.01602 gm/cm^3; 1 slug/ft^3 = 0.5154 gm/cm^3

Force

1 newton (nt) = 10^5 dynes = 0.1020 kgwt = 0.2248 lbwt
1 pound weight (lbwt) = 4.448 nt = 0.4536 kgwt = 32.17 poundals
1 kilogram weight (kgwt) = 2.205 lbwt = 9.807 nt
1 U.S. short ton = 2000 lbwt; 1 long ton = 2240 lbwt; 1 metric ton = 2205 lbwt

Energy

1 joule = 1 nt m = 10^7 ergs = 0.7376 ft lbwt = 0.2389 cal = 9.481×10^{-4} Btu
1 ft lbwt = 1.356 joules = 0.3239 cal = 1.285×10^{-3} Btu
1 calorie (cal) = 4.186 joules = 3.087 ft lbwt = 3.968×10^{-3} Btu
1 Btu (British thermal unit) = 778 ft lbwt = 1055 joules = 0.293 watt hr
1 kilowatt hour (kw hr) = 3.60×10^6 joules = 860.0 kcal = 3413 Btu
1 electron volt (ev) = 1.602×10^{-19} joule

Power

1 watt = 1 joule/sec = 10^7 ergs/sec = 0.2389 cal/sec
1 horsepower (hp) = 550 ft lbwt/sec = 33,000 ft lbwt/min = 745.7 watts
1 kilowatt (kw) = 1.341 hp = 737.6 ft lbwt/sec = 0.9483 Btu/sec

Pressure

1 nt/m^2 = 10 dynes/cm^2 = 9.869×10^{-6} atmosphere = 2.089×10^{-2} lbwt/ft^2
1 lbwt/in^2 = 6895 nt/m^2 = 5.171 cm mercury = 27.68 in. water
1 atmosphere (atm) = 1.013×10^5 nt/m^2 = 1.013×10^6 dynes/cm^2 = 14.70 lbwt/in^2
= 76 cm mercury = 406.8 in. water